Sensitivity Analysis in a Generic Multi-Attribute Decision Support System

Sixto Ríos-Insua, Antonio Jiménez and Alfonso Mateos
Department of Artificial Intelligence, Madrid Technical University
Campus de Montegancedo s/n, 28660 Boadilla de Monte, Madrid, SPAIN

Abstract. This paper describes several possible sensitivity analyses associated with a generic multi-attribute Decision Support System, which is capable of considering all the steps in the Decision Analysis cycle and is aimed at aiding decision-makers in the choice of the most preferred alternative. The system evaluates the set of alternatives through an additive multi-attribute utility function that allows for imprecise assignments concerning utilities and weights and uncertainty about consequences in terms of intervals instead of single values. The first sensitivity analysis is related to the traditional approach that can be used to answer "what if" questions. A series of graphical displays are an excellent aid in the decision-making process, because they give useful and important insight into the final ranking. Then, other complementary procedures are considered that exploit the imprecise inputs, generating results that provide insights into the model recommendations. This leads to the computation of non-dominated and potentially optimal alternatives, a weight stability interval assessment and simulation techniques that utilize Monte Carlo simulation methods allowing simultaneous weight changes.

Key words: Decision Support System, Imprecision, Multiattribute Utility, Sensitivity Analysis.

1. Introduction

The difficulty in solving complex real decision-making problems has recently led to the development of better decision support tools to deal with the increasingly involved difficulties. The tools to deal with such problems should be able to consider some of the basic aspects that arise in real problems, like the presence of multiple conflicting objectives, as well as imprecision concerning assignments. To deal with such complex situations, we have developed a decision support system (DSS) that allows to construct an objective tree with up to 200 objectives and 10 objective levels. The DSS is based on an additive multi-attribute utility model (Keeney and Raiffa, 1993), which allows for imprecision concerning the inputs. Thus, the process of assessing individual utility functions and constant scales is not very demanding and is, hence, less stressful for decision makers (DMs). Furthermore, we consider the situation where the consequences of the alternatives can be entered as ranges or intervals, modelled through continuous uniform distributions, instead of single values as an approach under certainty would demand, see (Mateos et al, 2001).

The starting point will be to establish a set of n attributes associated with the lowest-level objectives and denoted by $X_1,...,X_n$. Thus, the consequence of each alternative $S^q \in \mathcal{S}$, where $\mathcal{S}$ is the available alternatives set, is a vector of intervals $\left(\left[x_{1q}^L, x_{1q}^U\right],...,\left[x_{iq}^L, x_{1q}^U\right],...,\left[x_{nq}^L, x_{nq}^U\right]\right)$, where x_{iq}^L and x_{iq}^U are the lower (L) and upper (U) endpoints of the imprecise or uncertain consequence for attribute X_i, respectively. We also consider the possibility of substituting each interval by a single value given by the average $x_{iq}^P = \left(x_{iq}^L + x_{iq}^U\right)/2$ (P means precision), having then a precise consequence, if deemed appropriate by the DM.

Next, an analysis focused on judgemental inputs must be conducted to assess imprecise utility functions on attributes, which leads to a familiy of utility functions for each one with assessed extreme functions denoted by u_i^L and u_i^U, respectively (Jiménez et al, 2003). The evaluation process calls for precise utility functions in the problem-solving process, where u_i^P is the precise function, obtained by fitting natural cubic splines through the mid-points of the family of utility functions. Similarly, imprecise scaling factors or weights for each objective and attribute in the objective tree are assessed, obtaining the respective normalized intervals $\left[w_i^L, w_i^U\right]$, where w_i^P is the respective average normalized value. For the aggregation into a global utility, we assume the additive independence condition (Keeney and Raiffa, 1993) or an approximation (Raiffa, 1982; Stewart, 1996), which leads to the utility function of the form $u\left(S^q\right) = \sum_{i=1}^n k_i^P u_i^P\left(x_{iq}^P\right)$, where k_i^P is the precise weight obtained by multiplying the weights w_i^P in the path from the global objective of the hierarchy to attribute X_i. However, due to the possible imprecision in our approach concerning the consequences, the utility for each alternative S^q can given by a utility interval $\left[\sum_{i=1}^n k_i^L u_i^L\left(x_{iq}^L\right), \sum_{i=1}^n k_i^U u_i^U\left(x_{iq}^U\right)\right]$ (provided that the utility functions are increasing. If any utility function is decreasing, the change would be obvious).

Finally, we have recently developed several sensitivity analyses in the system, which will be explained in the following sections, because, as is well known, sensitivity analysis is an essential aid in any quantitative model to study the robustness of the final ranking of the alternatives. This methodology has been implemented on a PC-based DSS, where all the process-relevant information can be entered to help DMs arrive at the best or a satisfactory alternative (Jiménez et al, 2003).

2. Graphics-Supported Evaluation and Sensitivity Analysis

Once all the imprecise (or precise) information related to utilities, weights and consequences, denoted by $\boldsymbol{u} \in U$, $\boldsymbol{k} \in K$ and $\boldsymbol{x} \in X$, respectively, has been entered in the DSS, it computes an *Alternatives Classification*, which provides the ranking based on their precise values, as well as the respective associated utility intervals, see Figure 1.

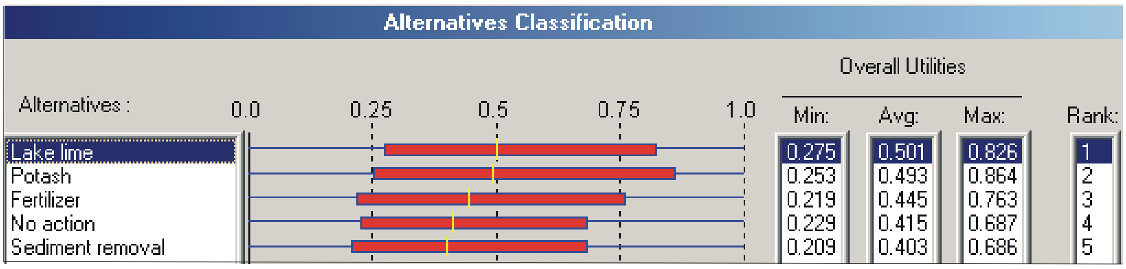


Figure 1. The utility intervals and the strategies ranked according to their precise utilities

This graph can be helpful for the DMs, because, besides providing the rank based on the precise utility, it gives the utility intervals that reflect the imprecision concerning the preferences and the uncertainty about the consequences. If the DM changes a utility, weight or consequence value, the system takes care of how these changes are propagated through the objective hierarchy and, automatically, recalculates the alternatives classification.
Another graphical aid is provided by *Weight Stability Intervals* for each one of the objectives or attributes in the hierarchy. Figure 2 shows the stability interval, where its weight can vary without affecting the overall ranking of the alternatives for a second level objective named, “Health impact”. The present value is 0.689, and we find that if this value is changed to another one outside the interval [0.681, 0.811], then the above ranking would change.

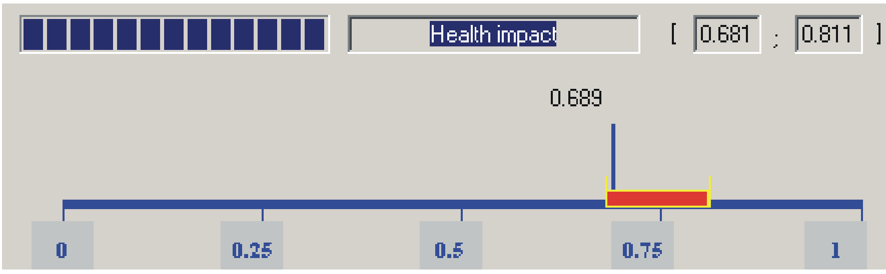


Figure 2. The stability weight interval for a given objective

Another display that is potentially useful for DMs is the *Stacked Bar Ranking*, which provides the average utility assigned to each alternative by breaking down the contribution for the different attributes in the utility bar, see Figure 3. Figure 3 also shows another graph named *Measure Utilities for Alternatives*, which displays the utility of the different attributes for a given alternative through bars whose width corresponds to the weight of the attribute in question.

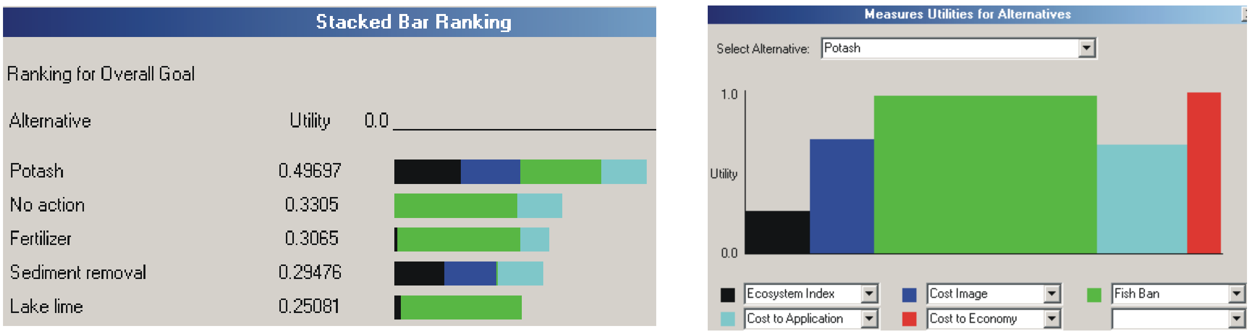


Figure 3. Displays of the Stacked Bar Ranking and Measure Utilities for Alternatives

Another display, Figure 4, shows the normalized weight intervals and normalized average weights associated with each attribute. This figure also shows the correlation between pairs of selected attributes for all the alternatives.
Finally, there is another possible display, named *Compare Alternatives Graph*, which compares two selected alternatives with respect to the utilities for each attribute in the model, thus providing how much more utility each alternative has with respect to the other for each attribute.

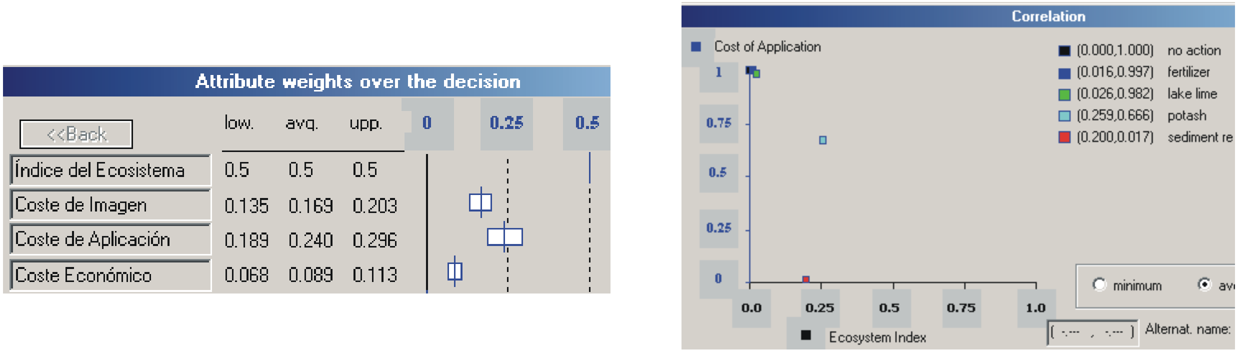


Figure 4. Display of the normalized interval and average weights, and correlation between alternatives

3. Nondominated and Potentially Optimal Alternatives

The ranges provided for utilities, weights and consequences are now used for simultaneous variation of all the data, taking advantage of advances in optimization theory to make it easier for the DM to choose an optimal solution. Thus, although it has been possible to output the ranking of alternatives based on their precise values, another possibility offered by the DSS is to exploit all the information about such ranges to better choose the most preferred alternative, as well as to reduce the set of alternatives of interest (Mateos et al, 2003). The main thrust will be, therefore, to order the alternatives in a Pareto sense. The DSS computes the *Non-Dominated* and *Potentially Optimal* alternatives by solving the optimization problems (for all the possible pairs of alternatives), respectively,

$$\begin{aligned} \min\ & f_{qp} = u(S^q) - u(S^p) \\ \text{s.t.}\ & \boldsymbol{u} \in U,\ \boldsymbol{k} \in K,\ \boldsymbol{x} \in X \end{aligned}$$

$$\begin{aligned} \min\ & f_p \\ \text{s.t.}\ & u(S^p) - u(S^q) + f_p \geq 0, \quad \forall p \neq q \\ & \boldsymbol{u} \in U,\ \boldsymbol{k} \in K,\ \boldsymbol{x} \in X, \end{aligned}$$

Figure 5 shows the non-dominated and the potentially optimal alternatives, reminding us that the optimal one is “Lake Liming”. Note that alternative “No Action” is not potentially optimal, and the set of alternatives to be considered by the DM has been reduced accordingly.

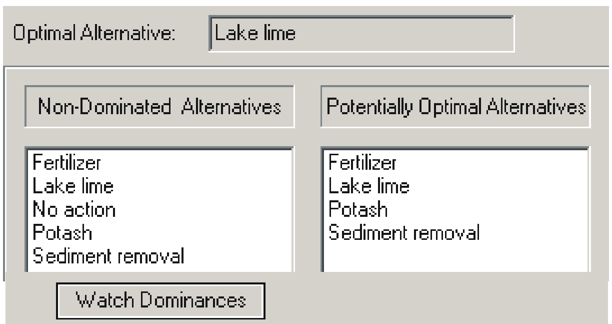


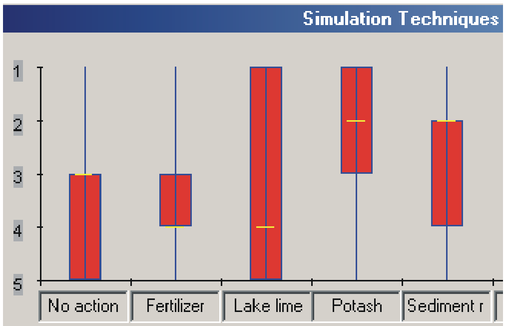
Figure 5. View of the non-dominated and the potentially optimal alternatives

4. Weight Simulation

The system also performs simulation techniques, which allows for simultaneous changes of the weights and generates results that can be easily analysed statistically to provide more insights into the multiattribute model recommendations (Butler et al, 1997). We propose selecting weights at random using a computer simulation program so that the results of many combinations of weights, including a complete ranking, can be explored efficiently. Three general classes of simulation are considered:

- *Random Weights*. As an extreme case, weights for attributes are generated completely at random. This approach implies no knowledge whatsoever of the relative importance of the attributes.
- *Rank Order Weights*. Randomly generating weights while preserving their attributes rank order places substantial restrictions on the domain of possible weights that are consistent with the DM's judgement of attribute importance. Therefore, the results from the rank order simulation may provide more meaningful results.
- *Response Distribution Weights*. The third type of sensitivity analysis using simulation recognizes that the weight assessment procedure is subject to variation. Now, attribute weights are randomly assigned values taking into account the interval weights provided by the DMs in the weights assignment.

As an aid, the system displays for each of the above cases a multiple boxplot for the alternatives and computes several statistics (mode, min, 25th percentile,..., mean, st. deviation) about the rankings of each alternative, see Figure 6. All this information can be useful for discarding some alternatives.



Statistics

	Mode	Min.	25th percentile	50th percent
No action	4	2	4.000	4.000
Fertilizer	3	1	3.000	3.000
Lake lime	1	1	1.000	1.000
Potash	2	1	1.000	2.000
Sediment removal	5	2	4.000	5.000

Figure 6. Multiple boxplot for the result of a simulation and some statistics about the ranking

Acknowledgements: This paper was supported by the Ministry of Science and Technology project DPI2001-3731.

References

Butler, J., Jia, J. and J. Dyer (1997); Simulation Techniques for the Sensitivity Analysis of Multicriteria Decision Models; European Journal of Operational Research, Vol. 103 (pp. 531-546)

Jiménez, A., Ríos-Insua, S. and A. Mateos (2003); A Decision Support System for Multiattribute Utility Evaluation based on Imprecise Assignments; Decision Support Systems (to appear)

Keeney, R.L. and H. Raiffa (1993); Decision with Multiple Objectives; Cambridge U.P.

Mateos, A., Jiménez, A. and S. Ríos-Insua (2003); Solving Dominance and Potential Optimality in Imprecise Multi-Attribute Additive Problems; Reliability Engineering and System Safety, Vol. 79, No. 2 (pp. 253-62)

Mateos, A., Ríos-Insua, S. and E. Gallego (2001); Postoptimal Analysis in a Multi-Attribute Decision Model for Restoring Contaminated Aquatic Ecosystems; Journal of the Operational Research Society, Vol. 52 (pp. 1-12)

Raiffa, H. (1982); The Art and Science of Negotiation; Harvard University Press; Cambridge.

Stewart, Th.J. (1996); Robustness of Additive Value Function Method in MCDM; Journal of Multi-Criteria Decision Analysis, Vol. 5 (pp. 301-309)